



(RESEARCH ARTICLE)



## Evaluating the performance of various Heuristic Algorithms to Solve Flow Shop Scheduling Problem with Fuzzy Processing Time

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World Journal of Advanced Research and Reviews, 2024, 24(01), 1922–1928

Publication history: Received on 19 August 2024 ; revised on 20 October 2024; accepted on 22 October 2024

Article DOI: <https://doi.org/10.30574/wjarr.2024.24.1.3211>

### Abstract

This paper evaluates the performance of selected heuristic algorithms, namely Palmer's and Campbell, Dudek and Smith (CDS) algorithm, Nawaz-Enscore-Ham (NEH), and SAI algorithms for minimizing the makespan in fuzzy flow shop scheduling problem (FSSP) with triangular fuzzy processing times. The algorithms are tested on sixteen benchmark problems with size up to 10 machines and 20 jobs using a MATLAB program. Computational results show that, despite its simplicity, the FNEH algorithm can provide the best results compared to other algorithms, regardless of the size of the problem.

**Keywords:** Flow shop scheduling problem; Fuzzy processing time; Heuristic; Makespan.

### 1. Introduction

The scheduling is considered one of the most important fields in operations research that plays an important role in in the planning and operation of a manufacturing and production systems. Due to the great development in production and manufacturing systems, the importance of the scheduling process has grown significantly, which has attracted the attention of many researchers to research into scheduling problems in the last decades. The flow shop scheduling problem (FSSP) is one of the most popular scheduling problems most widely studied scheduling problems, which can be defined and formulated using a variety of resources, tasks, and constraints [1].

The FSSP can be described as follows: a set of  $n$  jobs that have be scheduled on a set of  $m$  machines. Each job consists of a series of operations that to be processed on all machines in a predefined order. The objective is to determine the optimal sequence of  $n$  jobs to be processed on  $m$  machines to optimize one or more objectives. This problem was first introduced by Johnson to solve the problem of scheduling a flow store on two machines to minimize the total elapsed time [2]. After that, many researchers have been developed various deterministic algorithms to solve this problem in which processing times were taken as certain and known value. But in the real-world application, information about variables and parameters is often uncertain and imprecise. Although these algorithms have been successful in obtaining the optimal/near-optimal solution to the flow shop scheduling problem as mentioned by Eshim in [3], they are often difficult to apply in many real problems, due to incomplete knowledge or uncertainty about the data such as processing times, set-up time, idle time, and due-date etc. Therefore, to overcome this the problem, fuzzy set theory can be used to deal with uncertainty inherent in data related to scheduling problems that are difficult to determine precisely.

Recently, fuzzy set theory, introduced by Zadeh in 1965, has been considered as an effective mathematical tool for modeling uncertainties in scheduling fields [4]. Several studies on flow shop scheduling problems with fuzzy processing times and fuzzy due dates have developed for makespan minimization over the past fifty years. Ambika and Uthra [5] presented the branch and bound algorithm to obtain the job sequence with minimum makespan for three-machine flow

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shop problem with triangular fuzzy processing times. Another study developed a new heuristic algorithm for minimizing the makespan for two-machine flow shops with triangular fuzzy processing time. In this study, the results compared showed that the proposed algorithm performed the best performance in all cases [6]. Jadhav et al. [4] proposed an algorithm for solving flow-shop scheduling problem in fuzzy environment to optimize the total elapsed time. Job data are described by triangular and trapezoidal fuzzy number. The results have shown the efficiency and effectiveness of the proposed algorithm to find a job sequence with minimum makespan. Many scientists [7, 8] presented a method for flow shop scheduling problem with fuzzy processing time. Hong and Chuang [9] proposed the fuzzy Palmer algorithm for scheduling uncertain jobs in a flow shop with more than two machines. The conventional Palmer algorithm has been shown to be a special case of the fuzzy Palmer algorithm with special assigned membership functions. The fuzzy Palmer algorithm is then a feasible solution for both deterministic and uncertain flow shops with more than two machines. Kurniawan and Farizal [10] developed a new method for solving flow shop scheduling problems that based on the concept of the CDS and NEH methods namely, NEHLPD, NEHLPD1, NEHLPD2 and then tested it on random cases.

In a different study [1] applied matrix manipulation method in MATLAB to solve flow shop scheduling problem of  $n$  jobs on  $m$  machines under uncertain processing time. The problems have been considered for comparative analysis with Palmer's heuristic, CDS heuristic & NEH heuristic. The computational experiments showed that the proposed code and NEH heuristic outperforms over the other heuristics to find out the minimum makespan through an optimal sequence. Gupta et al. [11] used SAI method for solving sequencing problem when processing time of the machine is certain or uncertain in nature. The uncertainty in data is represented by triangular or trapezoidal fuzzy numbers. Yager's ranking function approach is used to convert these fuzzy numbers into a crisp at a prescribed value of  $\alpha$ .

The above-mentioned literature review indicates the continuous attention shown by the researchers in solving flow shop scheduling problems with the uncertain processing time using various methods. Therefore, in this study, the flow shop scheduling problem with triangular fuzzy processing time based on heuristic algorithms is presented. The main aim is to evaluate the performance of selected heuristic algorithms to find the best sequence jobs on machines to minimize the makespan.

### 1.1. Problem Statement

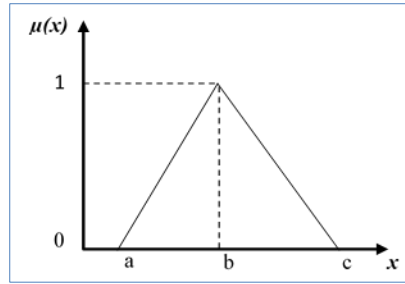
This paper deals with the flow shop scheduling problem under a fuzzy environment, so that the processing times of jobs are represented by triangular fuzzy numbers. Four heuristic algorithms, which include fuzzy Palmer's and fuzzy Campbell, Dudek and Smith (FCDS) algorithm, fuzzy Nawaz-Enscore-Ham (NEH), and fuzzy SAI algorithm are used. The aim is to evaluate the performance of heuristic algorithms to find the sequence of jobs to be processed on machines to minimize the makespan ( $\tilde{M}$ ). The assumptions of the present problem are:

- All the jobs and machines are available at time zero.
- Each job must be completed when started.
- To make job on a second machine, it must be completed on the first machine.
- Machines never break down and are available throughout the scheduling period.
- Machines may be idle.
- Setup times are known and are included in processing times.

As stated above that the processing time of jobs are represented by triangular fuzzy numbers and denoted by the formula  $(a, b, c)$ , where  $a, b, c$  are real numbers and its membership function is given by,

$$\mu_{\tilde{A}}(X) = \begin{cases} \frac{x-a}{b-a}, & \text{for } a \leq x \leq b \\ \frac{c-x}{c-b}, & \text{for } b \leq x \leq c \\ 0, & \text{elsewhere} \end{cases} \quad (1)$$

According to the above relation, the triangular membership functions of a fuzzy number are used to represent the fuzzy processing time of jobs on machines where the membership value reaches the highest point at 'b', while 'a' and 'c' denote the lower bound and upper bound of the processing time respectively. The values  $a, b$  and  $c$  are interpreted as pessimistic, moderate and optimistic values of the processing time, as shown in Figure 1.



**Figure 1** A triangular fuzzy number [4].

In order to convert the fuzzy numbers into crisp numbers, Average High-Ranking average (AHR) of the processing times for all the jobs on all machines are computed and then apply selected heuristic algorithms to find the best job sequence [4,5]. The formula of AHR was proposed by Yager's and can be calculated as follows:

$$h(A) = \frac{3b + c - a}{3} \quad (2)$$

Where a, b, c, is fuzzy processing time.

## 2. Heuristic Algorithms for FFSSP

In the section, the selected heuristic algorithms for solving fuzzy flow shop scheduling problem are presented.

### 2.1. Fuzzy Palmer's algorithm

D. S. Palmer's proposed a solution to the general  $(n, m)$   $n$  jobs and  $m$  machines problem by computing a slope index to give priorities to jobs to proceed from one machine to another and then sequencing the jobs in descending order of the slope index which give priority to jobs [9]. Jobs with processing times that tend to increase from machine to machine will receive higher priority, while jobs with processing times that tend to decrease from machine to machine will receive lower priority. The steps involved of fuzzy Palmer's algorithm to solve the FSSP with triangular fuzzy processing time are mentioned in paper [3].

### 2.2. Fuzzy Campbell, Dudek, and Smith (FCDS) algorithm

The Campbell Dudeck and Smith (CDS) method is the most important heuristic method for the makespan problem or the time to produce all jobs to completion in the flow shop scheduling problem[8].It is based on Johnson's two machine algorithm and creates a series of  $m-1$  auxiliary  $n$ -job, 2-machine problems from an original  $m$  machine problem, then each of the generated problems are solved using Johnson's algorithm to minimize the make-span in a deterministic flow shop problem as mentioned by Soltysova et al. [12]. The steps involved of FCDS algorithm to solve the FSSP with triangular fuzzy processing time are mentioned in paper [3].

### 3.3. Fuzzy Nawaz-Enscore-Ham(FNEH) Algorithm

This algorithm was developed by Muhammad Nawaz, E. Emory Ensore Jr., and Inyong Ham in 1983. The NEH method assumes that the job that has a larger total processing time for all machines must take precedence over the job with a smaller total processing time. NEH initializes the order of jobs in descending order based on the total processing time of each job. Then the partial sequence process is carried out, which is to determine the best order of each possible job position [6][13].

The steps of fuzzy NEH algorithm are as follows:

- Step1. Converting the fuzzy numbers into crisp numbers by computing average high ranking ( $A_{ij}$ ) of the processing times for all the jobs on all machines.
- Step 2: Add up all processing time for each job.
- Step 3: Sort jobs by number of processing times in descending order.
- Step 4: Choose the first two jobs from the list and create two partial sequences by swapping the positions of the two tasks. Compute the partial sequences' make-span values. The partial sequence with the shortest make-span is referred to as the incumbent sequence.

- Step 5: Select the next job from the work content list and assign it to all positions in the incumbent sequence. Determine the value of makespan for each sequence.
- Step 6: Keep the sequence with the shortest makespan as the incumbent sequence and reject all other sequences.
- Step 7: Stop if there are no jobs remaining in the work content list to be added to the incumbent sequence. Otherwise, go to step (5).

### 3.4. Fuzzy SAI Algorithm

This algorithm was proposed by Gupta et al. [14] and was used to determine the optimum solution for n jobs on m machines. In this study, we assume that the processing time of the machines for flow shop scheduling problems is uncertain. The procedure for this algorithm can be summarized as follows:

- Step1. Converting the fuzzy numbers for each job into crisp numbers by computing average high ranking.
- Step 2: Examine the jobs and select the least job processing time among all n jobs ( $j= 1,2,3,\dots,n$ )for each machine and then marked it with (-) sign.
- Step 3: Similarly, select the least processing time among all m machines ( $i= 1,2, 3,\dots,m$ ) for each job and then marked it with (+) sign.
- Step 4: Again examine the rows and columns of table, select the cell with ( $\mp$ )\_sign. Let the ( $\mp$ ) has occurred at cell which corresponds to  $i^{\text{th}}$  machine and  $j^{\text{th}}$  job. The  $j^{\text{th}}$  job is excluded from the table and is placed in the optimal job sequence.
- Step 5: Step 2to 4 are repeated until all the jobs are placed in the optimal job sequence. There may be a situation where a tie has occurred.
  - If ( $\mp$ ) occurs at more than one place, then the job with least processing time is selected and is placed in the optimal job sequence.
  - If ( $\mp$ ) occurs at more than one place and the processing time for the allocated jobs is same. Then the job which will process on the lower order positional machine is selected that is by ignoring the other higher order of machines.
- Step 7: Determine the value of make-span for the best sequence.

An experimental program was designed using MATLAB to code the previous heuristic algorithms, then it uses to solve considered benchmark problems in this study.

## 3. Computational Results

Computational experiments were performed on 16 benchmark problems with size up to 10 machines and 20 jobs, in order to evaluate the performance of the previous specified algorithms for solving fuzzy flow shop scheduling problem (FFSSP). These heuristic algorithms were used to obtain the best sequence on jobs on machines. The fuzzy makespan was calculated to use as a performance criterion between the four heuristic algorithms. The algorithms were coded in MATLAB over Intel(R2014a) core (TM) i3 CPU @ 2.20 GHZ computer with 2GB RAM. The result comparison is presented in Table 1.

**Table 1** Comparative Results of the Fuzzy Makespan

Problem size	FCDS Algorithm	Fuzzy Palmer's Algorithm	FNEH Algorithm	FSAI Algorithm
3×3	38,45,57	39,46,57	38,45,57	38,46,57
3×4	48,57,71	50,60,74	48,57,71	51,61,76
3×5	68,78,93	68,78,93	68,78,93	83,97,111
4×3	23,36,46	24,39,49	23,36,46	23,37,46
4×4	111,114,137	111,114,137	111,114,137	111,114,137
4×5	47,58,77	47,58,77	47,58,77	53,65,83
5×3	49,57,73	52,61,74	49,57,72	52,61,77

<b>5×4</b>	112,124,135	114,126,137	113,125,137	113,125,137
<b>5×5</b>	69,78,87	69,78,87	69,78,87	72,81,90
<b>6×3</b>	102,111,119	107,115,124	107,116,125	107,116,125
<b>6×4</b>	121,131,142	118,127,137	109,121,132	118,127,137
<b>6×5</b>	153,168,181	153,168,181	153,168,181	153,168,181
<b>10×5</b>	483,541,595	518,578,639	479,537,584	578,633,685
<b>10×10</b>	913,992,1060	907,990,1056	907,989,1058	957,1053,1116
<b>20×5</b>	1313,1384,1447	1338,1391,1444	1312,1375,1435	1544,1634,1689
<b>20×10</b>	1274,1410,1524	1122,1243,1352	1121,1227,1352	1369,1481,1593

Based on the above table, it can be seen that the results obtained using the FNEH algorithm were the most satisfactory and the best solution in 14 problems out of the sixteen benchmark problems, which represent (88%) of the obtained best solutions. While the FCDS algorithm reached the best solutions for (69%) of the obtained best solutions, (44%) and (25%) when applying the fuzzy palmer's algorithm and the fuzzy SAI algorithm, respectively.

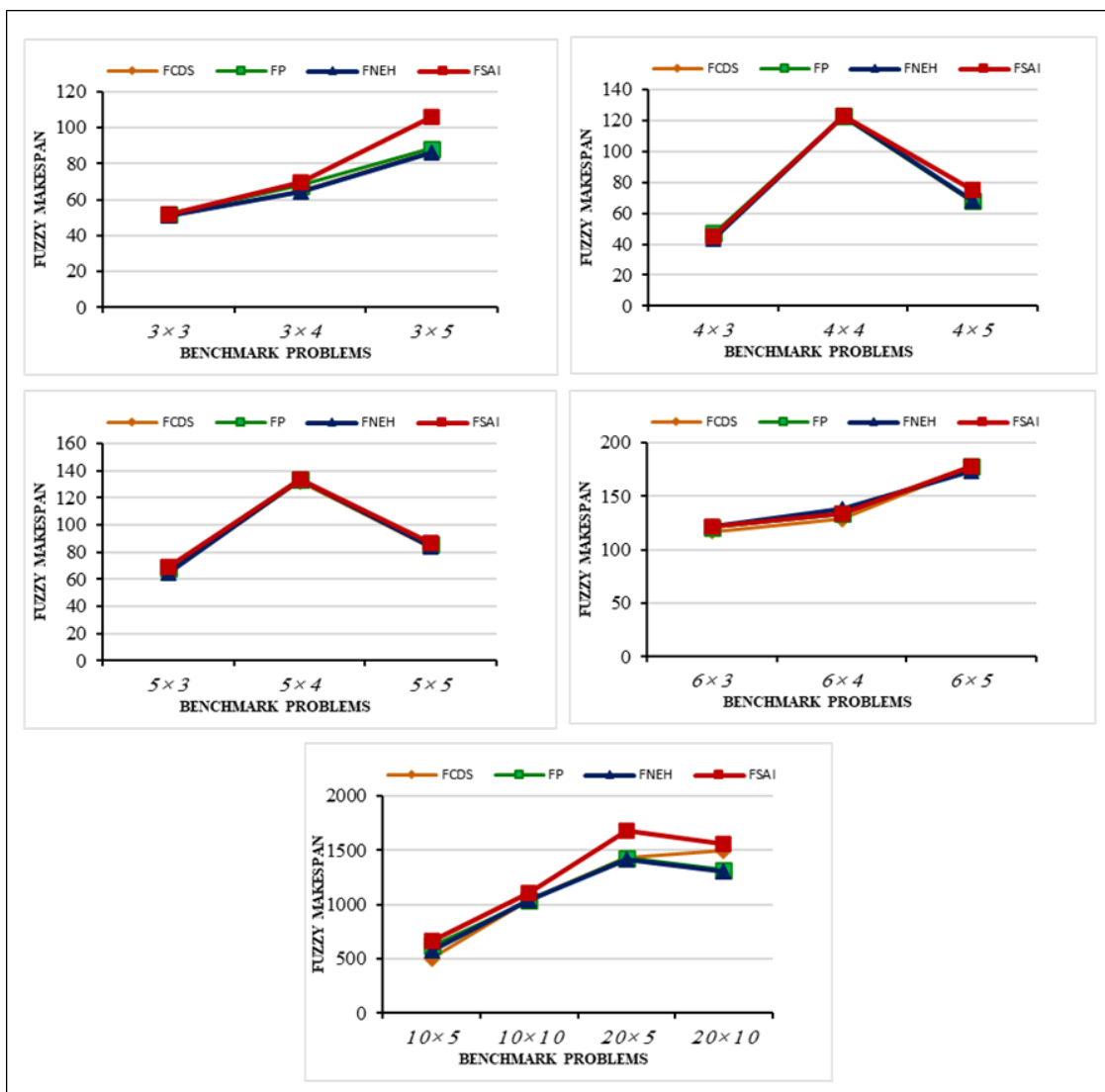
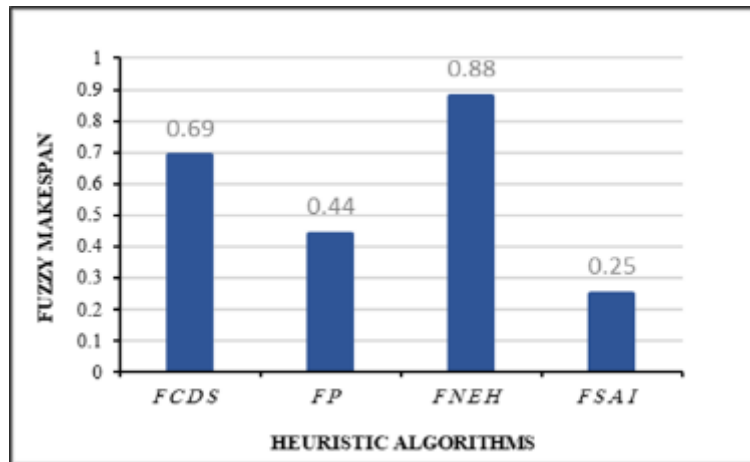


Figure 2 Comparative Results of Fuzzy Makespan for All Benchmark Problems

As it can be seen, the FCDS and FNEH algorithms achieved the same solutions in most benchmark problems for small-sized problems, but when the problem size increased, the fuzzy palmer's algorithm gave the best result compared to the FCDS and FSAI algorithms. Although the results obtained by the FSAI algorithm were achieved in four problems out of the sixteen considered benchmark problems, this algorithm has the advantage of no conversion of machines in the case of three or more machines. In summary, all of the results mentioned above show that FNEH algorithm was able to produce the best solutions in the most benchmark problems, regardless of the size of the problem, which can also be observed in Figure 2 and Figure 3, respectively.



**Figure 3** Comparison of the Best Algorithms Based on Fuzzy Makespan

## 5. Conclusion

The purpose of this study was to evaluate the performance of four heuristic algorithms to solve FFSSP to minimize the makespan. The experimental results of the 16 benchmark test problems validated that the FNEH algorithm can provide the best results compared to other algorithms, regardless of the size of the problem. In other words, there was a gradual growth in the makespan and it is evidently seen when the machine size starts increasing. For small sized-problems, the FNEH and FCDS algorithms behave similarly in producing the results, but when the problem size increased, the FNEH algorithm gave the best results compared to other algorithms in most benchmark problems. Then, the fuzzy palmer's algorithm was better than the FCDS and FSAI algorithms to give the best results. Based on these findings, a modified and hybrid FNEH algorithm can be suggested by combining it with meta-heuristic, like the Genetic Algorithm (GA), Simulated Annealing (SA), and Tabu Search (TS) as future work.

## References

- [1] Asaduzzaman, M., Saha, A., Hossain, M. S., & Mahmud, S. Minimizing Make span for Flow Shop Scheduling using Matrix Manipulation and Heuristic methods under processing uncertainty.
- [2] Asif, M. K. A., Alam, S. T., Jahan, S., & Arefin, M. R. (2022). An empirical analysis of exact algorithms for solving non-preemptive flow shop scheduling problem. *International journal of research in industrial engineering*, 11(3), 306-321.
- [3] Eshim, S. E. (2024). A Comparative Study of Heuristic Algorithms to Solve Flow Shop Scheduling Problem with Fuzzy Processing Time. *Int. J. Sci. Eng. Appl*, 13, 093-096.
- [4] Jadhav, V. S., & Jadhav, O. S. (2019). Solving flow-shop scheduling problem to minimize total elapsed time using fuzzy approach. *International Journal of Statistics and Applied Mathematics*, 4(5), 130-133.
- [5] Ambika, G., & Uthra, G. (2014). Branch and bound technique in flow shop scheduling using fuzzy processing times. *Annals of Pure and Applied Mathematics*, 8(2), 37-42.
- [6] Hejari, S. R., Emami, S., & Arkan, A. (2009). A Heuristic algorithm for minimizing the expected make span in two machine flow shops with fuzzy processing time. *Journal of uncertain systems*, 3(2), 114-122.
- [7] Mccahon, C. S. and Stanley, L. E. (1992). Fuzzy job sequencing for a flow shop. *European Journal of Operational Research*, Vol. 62, No.3, pp. 294-301.

- [8] Tsujimura, Y., Park, S.H., Chang, I. S., and Gen, M. (1993). An effective method for solving flow shop scheduling problems with fuzzy processing times. *Computers and Industrial Engineering*, Vol. 25, No., pp.239-242.
- [9] Hong, T. P., & Chuang, T. N. (1999). Fuzzy palmer Scheduling for Flow Shops with More than Two Machines. *J. Inf. Sci. Eng.*, 15(3), 397-406.
- [10] Kurniawan, L. A., & Farizal, F. Development of Flow Shop Scheduling Method to Minimize Makespan Based on Nawaz Enscore Ham (NEH) & Campbell Dudek and Smith (CDS) Method.
- [11] Gupta, S., Ali, I., & Ahmed, A. (2017). SAI method for solving job shop sequencing problem under certain and uncertain environment. *Sri Lankan Journal of Applied Statistics*, 18(3).
- [12] Soltysova, Z., Semanco, P., & Modrak, J. (2019). Exploring heuristic techniques for flow shop scheduling. *Management and Production Engineering Review*, 10(3).
- [13] Alharkan, I. M. (2005). Algorithms for sequencing and scheduling. Industrial Engineering Department, King Saud University, Riyadh, Saudi Arabia.
- [14] Gupta, S., Ali, I., & Ahmed, A. (2016). A new algorithm for solving job shop sequencing problem. *International journal for computer science engineering*, 5(2), 93-100.